Lecture 6: Non-Parametric Hypothesis Testing

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Goals

- Recap combinations/binomial distribution
- Review parametric tests
- Non-parametric tests: Sign test, Wilcoxon signed-rank, Fisher's exact test

Section 1

Recap

Recapping Algebra: Binomial Probability

Coin flip 10 times. $P(\text{exactly 7 heads})$?

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$P(X = 7) = \binom{10}{7} (0.5)^7 (0.5)^3 = 0.117$$

Biased Coin

If $p = 0.6$:

$$P(X = 7) = \binom{10}{7}(0.6)^7(0.4)^3 = 0.215$$

Hypothesis Testing (Recap)

- Examine H_0 (null) and H_A (alternative)
- Mutually exclusive and exhaustive
- Calculate test statistic $\rightarrow$ compare to null distribution $\rightarrow$ p-value

P-values (Recap)

The **p-value** is the probability of observing a test statistic as extreme or more extreme than the observed value, **assuming H_0 is true**.

Parametric vs. Non-parametric Tests

Parametric:

- Relies on theoretical distributions + normality assumption
- Examples: t-test, ANOVA, F-test

Non-parametric:

- Does not assume a specific distribution
- Uses ranks or signs instead of raw values
- Examples: Sign test, Wilcoxon, Mann-Whitney, Fisher's exact

t-Statistic (Recap)

$$t = \frac{\bar{x} - \mu_0}{\hat{\sigma}/\sqrt{n}} \sim t_{n-1}$$

Student's t-Distribution (Recap)

- Heavier tails than normal
- Approaches normal as df $\rightarrow \infty$
- Accounts for uncertainty in estimating σ

Two-Sample t-Tests (Recap)

Paired: $d_i = x_i - y_i$, $t = \bar{d}/(\hat{\sigma}_d/\sqrt{n})$

Unpaired (pooled): $t = \frac{\bar{X}-\bar{Y}}{\hat{\sigma}_p\sqrt{1/n_X+1/n_Y}}$

Key Assumptions in t-Test

1. Random sampling
2. Independence
3. Homogeneity of variances
4. **Normality**

What if normality is violated? $\rightarrow$ Non-parametric tests

Section 2

Non-Parametric Tests

What Do We Mean by Non-parametric?

Distribution-free methods — do not assume a specific parametric form for the underlying population distribution.

- More **robust** to violations of assumptions
- Work on **ordinal** data (ranks)
- Generally have **less power** than parametric tests when assumptions are met

The Sign Test

Non-parametric equivalent of one-sample t-test

$$H_0 : M = M_0 \quad \text{(median equals } M_0\text{)}$$

Test statistic S: count the number of observations $> M_0$.

Under H_0: $S \sim \text{Binomial}(n, 0.5)$

Sign Test — Procedure

1. Compute $x_i - M_0$ for each observation
2. Count S^+ = number of positive differences
3. Under H_0: $S^+ \sim$ Binomial$(n, 0.5)$
4. P-value: $P(S^+ \geq s^+_{\text{obs}})$ (one-sided) or $2 \times \min(P(S^+ \geq s^+), P(S^+ \leq s^+))$ (two-sided)

Sign Test — Two-sided

For a two-sided test at level α:

$$\text{p-value} = 2 \times P(S^+ \geq s^+_{\text{obs}} \mid H_0)$$

if $s^+_{\text{obs}} > n/2$, or the symmetric version otherwise.

Assumptions of the Sign Test

- Observations are **independent**
- Population is **continuous** (ties are discarded)
- Only uses the **signs** of differences — ignores magnitudes
- Very simple but **low power** (wastes information about effect sizes)

Section 3

Wilcoxon Signed-Rank Test

Wilcoxon Signed-Rank Test

Uses **magnitudes** as well as signs — more powerful than the sign test.

$$H_0 : M = M_0$$

1. Compute $|x_i - M_0|$ and $\text{sgn}(x_i - M_0)$
2. Rank the $|x_i - M_0|$ values
3. $W = \sum R_i \cdot \text{sgn}(x_i - M_0)$

Wilcoxon — Worked Example

Transcript level in 10 cells, $M_0 = 20.5$:

x_i	$x_i - M_0$	$\|x_i - M_0\|$	Rank	Signed Rank
17.5	-3.0	3.0	8	-8
18.3	-2.2	2.2	7	-7
19.2	-1.3	1.3	5	-5
20.1	-0.4	0.4	2	-2
20.8	0.3	0.3	1	1
21.0	0.5	0.5	3	3
22.4	1.9	1.9	6	6
23.1	2.6	2.6	9	-9
24.0	3.5	3.5	10	10
25.5	5.0	5.0	11	11

Wilcoxon — Test Statistic

$$W^+ = \sum_{i:x_i > M_0} R_i \qquad W^- = \sum_{i:x_i < M_0} R_i$$

$$W = \min(W^+, W^-)$$

Compare to critical values from the Wilcoxon distribution table, or use the normal approximation for large n.

Distribution of W

- For small n: use exact tables
- For large n: normal approximation

$$E[W] = \frac{n(n+1)}{4}, \quad \text{Var}(W) = \frac{n(n+1)(2n+1)}{24}$$

$$Z = \frac{W - E[W]}{\sqrt{\text{Var}(W)}} \approx N(0,1)$$

Wilcoxon Summary

- Can compare single sample to a hypothesized median
- Can compare **paired** medians (apply to differences)
- More powerful than Sign test (uses magnitudes)
- Still non-parametric (no normality assumption)

Section 4

Other Non-parametric Tests

Fisher's Exact Test

For 2×2 contingency tables — tests independence of two categorical variables.

	Category 1	Category 2	Total
Group A	a	b	$a+b$
Group B	c	d	$c+d$
Total	$a+c$	$b+d$	n

Fisher's Exact Test — Probability

Under H_0 (independence), the probability of observing this table is given by the **hypergeometric distribution**:

$$P = \frac{\binom{a+b}{a}\binom{c+d}{c}}{\binom{n}{a+c}}$$

Sum probabilities of all tables as extreme or more extreme to get the p-value.

Chi-squared Test

For larger contingency tables, use the **parametric** χ^2 test:

$$\chi^2 = \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

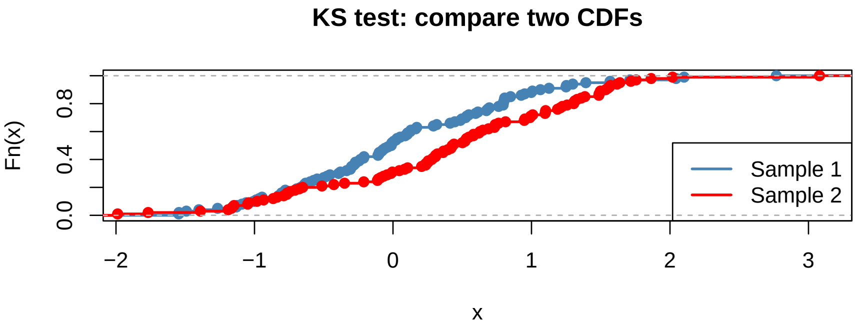
where $E_{ij} = \frac{(\text{row total}_i)(\text{col total}_j)}{n}$

Relies on χ^2 distribution — requires sufficient expected counts (≥ 5).

Kolmogorov-Smirnov Test

Tests whether two samples come from the same distribution.

$$D = \max_x |F_1(x) - F_2(x)|$$



Non-parametric Tests — Summary

Test	Parametric equivalent	Use case
Sign test	One-sample t-test	Median test
Wilcoxon signed-rank	Paired t-test	Paired medians
Mann-Whitney U	Unpaired t-test	Two independent groups
Fisher's exact	χ^2 test	2×2 contingency
KS test	—	Compare distributions

Trade-off: Robust to violations of normality, but decreased power when parametric assumptions hold.